Beauty by numbers

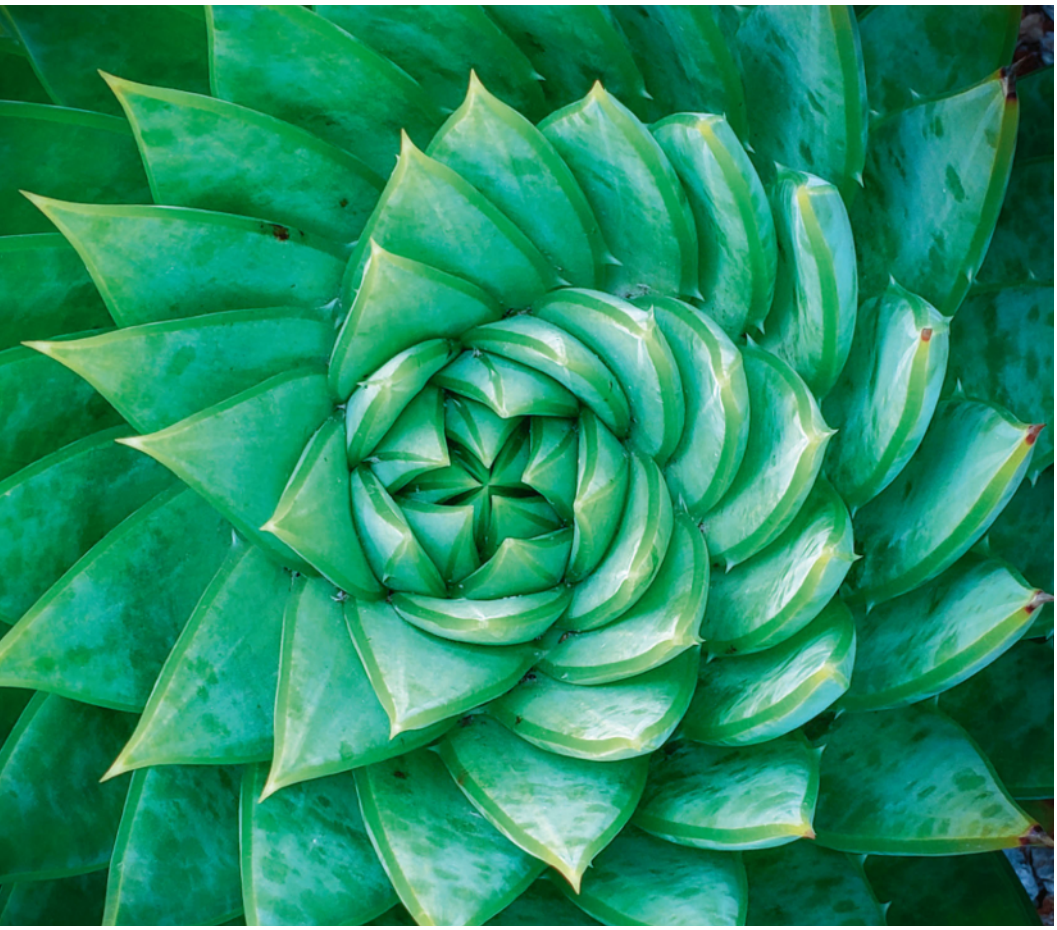
What do spiders' webs, snowflakes and snail shells have in common? They all contain fractals: Nature's exquisite, endlessly repeating mathematical pattern. **Deborah Nicholls-Lee** unpicks their complex geometry

A FRACTAL, according to Franco-Polish mathematician Benoît Mandelbrot, is 'a way of seeing infinity'. Simply put, it is a pattern that appears to self-replicate indefinitely, constantly reproducing an imitation of the previous shape. The term dates back to 1975, when Mandelbrot—to whom it is credited—applied the mathematics of theoretical fractional dimensions to the geometric patterning found in the natural world. Fractals are everywhere, even within the human body: our nervous system, blood vessels and the structure of our brain and lungs are all examples.

> 'Fractals are even in the human body: our nervous system, the structure of our brain'

The archetypal fractal takes a form that is a blueprint for the smaller elements within it. The trunk and boughs of a tree, for example, echo the forked forms within the tree as the branches split into twigs, which, in turn, throw off shiny new shoots. The flat-planed geometry that we often use to describe forms is inadequate when it comes to conceptualising them: 'Clouds are not spheres, mountains are not cones, coastlines are not circles and bark is not smooth,' Mandelbrot asserted. →

Ever-increasing circles: the vibrant heart of the ornamental pink flowering leaf cabbage



Above: **As the spiral aloe grows, the plant fans out to maximise the sunlight for each leaf.**
Below: **Focal point: the eye is drawn into the mesmerising swirl of seeds in a sunflower**

Fractals are all around us throughout the year, but, as the cold starts to bite, they become that much more obvious. Acorns in lacy caps litter the ground, pine cones are stuffed in children's pockets and the most flamboyant edible fractal, Romanesco broccoli, is ready to be harvested as autumn arrives. With the first frosts come intricate ice fractals, impossibly delicate and robustly infinite in equal measure.

Sometimes, it's only when we take an aerial view that fractals make themselves known. The saltmarshes of Cornwall's Camel estuary, for example, feature twisting rivulets of water that branch off themselves time and time again, clawing their way through this lush marine conservation zone (*'This is England', October 2*). At ground level, the overarching effect of these is imperceptible—seen from the air, however, the landscape's full fractal glory becomes immediately and gloriously clear. Meanwhile, looking across the country to East Anglia, the tide etches interminable markings like a tiger's stripes into the mudflats of the north Norfolk coast.

> *'Seeking out* patterns helps us make sense of the world around us'

Occasionally, the reverse is true and it's when we turn our gaze upwards that the fractals reveal themselves. Contemplating cauliflower-like cumulonimbus and cumulus cloud formations is a peaceful pastime, whereas forked lightning provides one of the planet's most dramatic visual displays of a fractal in action. Below ground, another huge network of fractals is at work in the form of root systems, water channels and plant runners, all of which combine to create a hidden web that helps sustain the teeming life above.

The stuff of art, then (of which more later)—but also of science. 'The perfect fractal is self repeating to infinity, mathematically,' clarifies Chris Whitewoods, a plant developmental biologist and the research group leader at the Sainsbury Laboratory in Cambridge. However, it's a little more complex than that when the mathematics is applied to Nature. Most plants make this self-iterative, repeating structure, he goes on to explain, but each iteration will differ slightly from the one before: 'No plant will make a perfect fractal.'

What actually causes natural forms to grow in this way? Ultimately, Dr Whitewoods explains, it's 'to do with the fact that they can't move'. 'They are literally rooted to the spot, which causes a lot of problems. If we were

about to get eaten, we would run away, but plants can't do that—so they move towards something they like or want, or move away from something that's not good for them, through development and growth. One way they do this is by growing from meristems—little groups of stem cells at their tips—which creates repeated patterns.'

Think of a spiral-shaped succulent, such as an *Aloe polyphylla* or the pretty fronds of the common fern. Their fractal look, scientifically speaking, is 'a snapshot of development: you've got leaves at all these different stages, all constantly being made and all constantly growing'. It's clever in more ways than one. 'The spiral pattern spaces out the leaves nicely, so that you're not getting overlap, which means each leaf can get more sunlight,' notes Dr Whitewoods. Hormones can also play a role, releasing a signal that inhibits growth. 'Plants are actively controlling the pattern of their branching.'

What makes fractals so appealing to our eyes? Patterns aren't only useful to growing organisms—on a neuroscientific level, they are helpful and even therapeutic to behold, says Pembrokeshire-based clinical psychologist and ecotherapist Suzi Tarrant. 'Being able to create patterns from the sensory data we receive in our brains is essential for our survival; otherwise, we would be overwhelmed by the complexity of what we're experiencing moment by moment,' she observes. 'So *seeking out* patterns helps us make sense of the world around us.' In a frequently chaotic and unstable-seeming milieu, patterns offer us, on a subliminal level, 'a sense of predictability, regularity and logic', which, in turn, creates reassuring feelings of safety.

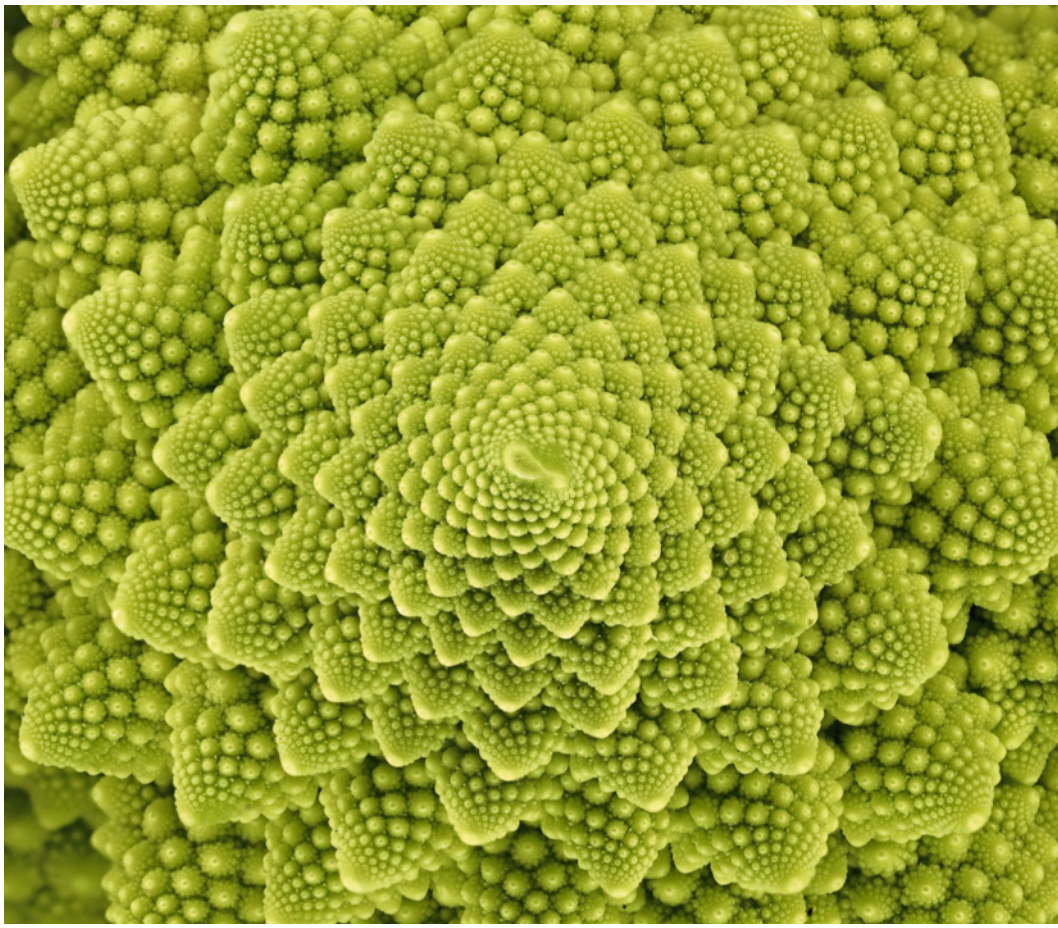
Fractal forms, in particular, have been found by scientists to generate positive emotions →

Edible forms

Broccoli
One of the most familiar fractals is the tree and this vegetable is a miniature version, branching out in shapes that mirror the broccoli head as a whole

Artichoke
There's some debate as to whether this technically counts as a fractal, but, when viewed from above, an artichoke's spiralling petal formation appears to take a fractal form. Rather like a Russian doll, the shape of the tasty heart at its centre mimics the globe itself

Red cabbage
This brassica might look unspectacular from the outside, but slice it in half across the girth and a dramatic purple-and-white fractal appears



Above: **The red cabbage in cross section reveals an intricate interweaving of patterns.**
Below: **Flamboyant fractals: a whorled Romanesco broccoli is almost too exquisite to eat**

Cool as ice: the sparkling snowflake is among the most fleeting of fractals, but these wonders of winter live long in the memory

in humans. 'Looking at fractals can increase our production of alpha waves in the brain, which are associated with a calmer and more relaxed state,' Ms Tarrant attests, stating that they can also help to regulate our levels of cortisol, a stress hormone. Ongoing research at the University of Oregon in the US supports this. One study detected a 60% improvement rate in recovery from stress among participants exposed to fractal images, whereas non-fractal images actually increased their level of stress. In 2016, one of the leaders of the study observed that 'as society increasingly surrounds itself with urban landscapes, people risk disconnecting from this natural stress-reducer'. Seeking out fractals can help to offset this. 'They evoke a sense of beauty and wonder and awe,' Ms Tarrant agrees. 'They act as a bridge between us and the rest of Nature.'

> 'They evoke a sense of beauty and wonder; a bridge between us and the rest of Nature'

With this in mind, it makes perfect sense that many of the most enduringly popular artworks riff on fractals, knowingly or otherwise. The swirling sky, stars and sun of Vincent van Gogh's *The Starry Night* (1889), for example, have a fractal quality, as does the cypress tree in the foreground, its flame-like branches mirroring its overall form. The Dutch painter also saw this infinite patterning in the sea and would return to paint it on numerous occasions. In a letter to his brother, Theo, in 1877, he describes the ocean as 'an image of eternity' that sets its 'ever-moving stability against the forces of time'.

Fifty years earlier, Katsushika Hokusai's woodblock print *The Great Wave off Kanagawa* (1831) was already foaming up in spiky fractals. The artist spoke of his determination to learn 'the pattern of Nature' and to 'cut my way deeply into the mystery of life'. A century on, M. C. Escher's spiralling, tessellating forms, such as his woodcut of swimming fish, *Path of Life I* (1958), testify to his mission to express eternity in art. 'I am interested in patterns with "motifs" getting smaller and smaller till they reach the limit of infinite smallness,' he wrote in 1958. Even the seemingly haphazard 'poured' works of Jackson Pollock, with their frenetic splashes of paint, have been revealed to be fractals, perhaps explaining their wide appeal.

Returning to the present day, Ms Tarrant's own interest in Nature's extraordinary fractals recently saw her make a trip to her local delicatessen in the hope of sourcing a Romanesco broccoli. Although the mission was completed, the planned outcome was not. 'I couldn't eat it because it looked so beautiful,' she admits. 'I was so captivated by what it looked like.'

Fractal facts

- The term 'fractal' originates from the Latin *fractus*, meaning 'broken' or 'shattered'. It speaks to the 'broken', divergent elements, the role of which in a structure can be understood when viewed as part of a whole
- The first fractal molecule was discovered earlier this year by a team of researchers led by groups from the Max Planck Institute and the Philipps University in Marburg, Germany. The microbial enzyme, found in a cyanobacterium, is a protein called citrate synthase, which arranges itself in the fractal Sierpinski triangle formation
- In 2015, this same tetrahedral formation made it into Guinness World Records when mathematician Caroline Ainslie teamed up with 11 balloon artists to create the world's largest 3D fractal. The complex construction featured 1,600 balloons and stood more than 7ft tall

Getty; Alamy